COST OF CAPITAL AND INVESTMENT IN A NON-INTEREST ECONOMY

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It has been contended that Islamic banks and financial institutions continue to use the interest rate mechanism as a benchmark for procurement and placement of funds because in the absence of an interest rate mechanism there is no benchmark against which the cost of capital can be measured and the efficiency of investment projects evaluated. This paper proposes a method by which the cost of capital can be measured without resort to a fixed and predetermined interest rate. The suggested procedure is simple and is based on the well-known Tobin’s q and can be used in the private as well as in the public sector to obtain benchmarks in reference to which investment decisions can be made.

1. INTRODUCTION

In an economy in which interest rates form the basis for financial intermediation, the market rate of interest provides a means for calculating the cost of capital and a benchmark against which the internal rate of return or the marginal efficiency of capital of an investment project can be measured. When the interest rate is removed from an economy, it has been contended that there is no mechanism by which the cost of capital can be measured, and, consequently, the efficiency of investment projects cannot be evaluated. As shown elsewhere (Khan and Mirakhor, 1988), in an Islamic economy, there is a rate of return in the financial market based on and closely related to the rate of return in the real sector of the economy that can serve as a benchmark for investment decisions. In the transition, however, there are methods by which cost of capital can be measured without resort to a fixed and predetermined rate of interest. Such a measure can be used in the private sector as well as the public sector to obtain benchmarks in reference to which investment decisions are made.

The purpose of this paper is to suggest a procedure by which the cost of capital can be measured in the absence of a market rate of interest. The suggested procedure utilizes Tobin’s q in calculating the cost of capital in an economy where debt instruments have been removed. In the absence of a fixed and predetermined rate of interest, equity financing becomes the only source of financial capital, and as such, the economy’s financial system becomes equity-based. In the capital

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market in such an economy, there are two groups of “investors”. These are the portfolio or financial investors, on the one hand, who enter the capital market seeking outlets for their savings or surplus funds, and the entrepreneurs, on the other hand, who enter the capital market seeking funds in order to carry out their investment plans. In an economy without debt instruments, the stock of physical capital is valued in the market for equities, and a relationship between the supply price of capital and the rate at which the shareholders discount an expected future stream of earnings can be derived. The evaluation of physical capital, resulting from entrepreneur’s investment plans, via equity markets provides the essential input to the process of interaction of suppliers of funds and entrepreneurs, and it is this interaction which determines the supply price of capital.

Tobin defines the supply price of capital as “the rate of return that the community of wealth-owners require in order to absorb the existing capital stock (valued at current prices), no more no less, into their portfolios and balance sheets”. 1 The incentive for companies to invest will depend on prospective profitability relative to the cost of capital. The rate of return is the ratio of profits to physical capital employed valued at replacement cost, while the corresponding cost of capital is the ratio of the same profit figure to the financial valuation of companies. Thus, relative profitability is simply the ratio of the financial valuation to the replacement cost of capital. This ratio can be seen as measuring the divergence between the demand and supply prices of capital goods. On this basis, investment should be expected to occur when the demand price, as reflected in financial valuations, exceeds the supply price, as measured by the replacement cost of physical capital.

It is conventional practice to estimate the return on shareholders’ equity by combining dividend receipts with retained earnings. In Islamic finance, this treatment could be extended to all profit and loss sharing activities. In this way, profitability on physical assets can be compared with return to all the savers who have financed the investment, including the ordinary shareholders.

2. AN ALTERNATIVE TO THE RATE OF RETURN IN MEASURING PROFITABILITY

The q theory of investment, which relates investment to the ratio of market to replacement value of capital, has attracted considerable attention. Yoshikawa (1980), Summers (1981), and Hayashi (1982) show that under certain conditions the rate of investment of a share value-maximizing firm is a function of q. Marginal q is the ratio of the market value of an additional unit of capital to its replacement cost. However, what is observed is average q, namely the ratio of the market value of existing capital to its replacement cost. Marginal q is the

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fundamental determinant of investment because it shows how much an increase in market value accompanies a dollar’s worth of investment, while the actual market value of a firm reflects the profitability of existing capital.

Based on a cash flow model in which the firm faces convex costs in adjusting its capital stock, it has been shown that physical investment is determined by marginal $q$, defined as the ratio of the discounted future revenues from an additional unit of capital (i.e., the shadow price of capital) to its net-of-tax purchase price [Mussa (1977), Abel (1979)]. Critical to the operational usefulness of this model is that unobservable marginal $q$ must be related to observable average $q$, defined as the ratio of financial to physical capital. The conditions under which this connection can be drawn have been established in Hayashi (1982).

Tobin’s $q$, the ratio of a firm’s market value to its replacement cost, has several advantages over the rate of return as a measures of a firm’s profitability. The numerator of $q$, the firm’s market value, reflects the firm’s expected future profits, while the accounting rate of return measures only past profits. Furthermore, a firm’s market value also is influenced by the variance of expected profits, so $q$ includes an automatic adjustment for risk. Finally, because the denominator of $q$ is a firm’s replacement value and not its book value, $q$ is less sensitive to inflation than the accounting rate of return.\(^2\) Accounting estimates of $q$ have smaller average errors than the accounting rate of return.

With regard to profit rates, while profit rates can be compared to an estimate of a risk-adjusted required return, the results might be sensitive to the choice of an asset pricing model. Disequilibrium rents cause less fluctuation in market value than in profits; also high profits may be a return to risk rather than evidence of profitable investment opportunities. No matter how risk premiums are actually determined in the capital markets, $q$ should equal unity if profits are just high enough to compensate the shareholders for risk. Moreover, profits are small compared to revenues and costs and, in this regard, measured profits are highly sensitive to errors in measuring revenues and costs. For example, if costs are ten times as large as profits, then a one percent error in measuring costs causes profits to be mismeasured by 10 percent. Because, accounting rules do not adequately capture the effects of inflation and accounting depreciation generally differs from economic depreciation, the errors in measuring profit rates are probably large (Fisher and McGowan, 1983). In contrast, $q$ is much less sensitive to errors in

\(^2\) As with accounting rates of return, however, the $q$ ratio is also open to objections relating to the evaluation of a firm’s capital assets. The replacement cost of a firm’s assets, which is the denominator of $q$, often excludes any measure of the firm’s intangible assets and includes a measure of depreciated tangible assets that is calculated using depreciation schedules that do not adequately reflect true economic depreciation. Thus the exclusion of intangibles from the firm’s asset base and the use of inadequate depreciation schedules, which are the major objections to the accounting rate of return, also may cause errors in $q$.\)
measuring its components. If the capital stock is overestimated by one percent, q is underestimated by only one percent.

3. FINANCIAL VALUATION AND THE Q THEORY OF INVESTMENT

The cost of capital is the cost to the company of the finance needed to acquire the physical capital. It can be argued that neither interest rates nor the cost of equity finance alone provide a good measure of the cost of capital, because the way in which changes in the proportions of debt and equity in a company’s balance sheet affect the riskiness of their respective yields, and hence their market valuation. Assume that a company has some long-term fixed-interest debt outstanding; that its current dividends and current and future real profits (before interest) are given; and the real cost of equity capital, i.e., the rate at which the market discounts expected real returns to shareholders when determining the share price, is fixed. An increase in the expected rate of inflation would reduce the real value of future interest payments due on the outstanding long-term loans. This would raise the present value of future equity earnings as, by assumption, expected future real profits are unchanged while future real prior charges fall. The price of ordinary shares should, therefore, tend to rise so that, as neither current dividends nor earnings have changed, the dividend and earnings yields will each be reduced - even though, by assumption, the real cost of equity capital is unchanged. Thus, unanticipated inflation transfers real wealth from a company’s debenture holders to its equity holder. To the extent that inflation tends to make debt seem more expensive than it really is, and equity seem cheaper, this requires that the root of two types of capital should be combined in an interest-based economy. However, in an Islamic economy there would be no predetermined rate of return, and the equity market would provide a measure of the cost of capital. Though there are a number of models of q, here we present the simplest version formulated by Hayashi (1982).3

Consider a firm with the following production function F

\[ Q = F(L,K), \]

where Q is output, L is labor employed and K is the stock of capital. F (·) is assumed to be a concave function of K and L and is twice continuously differentiable with

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3 See Barro and Sala-i-Martin (1995), pp. 119-125 for a more recent formulation.
The firm is organized on the basis of project and loss sharing with a wage rate \( w \) and price of output \( p \) and a rate of return \( \rho \) required by shareholders on equity capital, the firm can find an optimum level of employment by maximizing

\[
\int_0^\infty (pQ - wL)e^{-r_0} dt
\]

(2)

The necessary condition is given by (3)

\[
p\left(\frac{\delta Q}{\delta L}\right) = w \quad \text{for all } t
\]

(3)

Assuming (1) to be a constant return to scale production function, (3) can be written as:

\[
f(k) - f'(k)k = w/p
\]

(4)

where \( k = K/L \) and \( f(k) = f(K/L, 1) \). With these modifications, the expected rate of profit, \( r \), is:

\[
r = f'(k)
\]

(5)

and, with optimum employment decision, the market value of the firm (2) becomes:
\[
\int_0^\infty prKe^{-\rho t} dt = prK / \rho
\]

(6)

Note that, given the definitions of \( p, r, K, \) and \( \rho \), (6) is the market value of the firm’s capital stock. The cost of replacing the same capital stock is \( pK \). Recalling that Tobin’s \( q \) is defined as the ratio of market value of the firm’s capital stock to its replacement cost, dividing (6) by \( pK \) yields:

\[
q = \frac{r}{\rho}
\]

(7)

which indicates that investment will take place until the rate of return the shareholders expect equals the rate of profit expected from that investment. Denoting the market value of the capital stock as \( V \) and its replacement cost as \( C \), (7) becomes:

\[
q = \frac{r}{\rho} = \frac{V}{C}
\]

(8)

In making investment decision, the firm must decide how rapidly it can add to the stock of capital since in doing so, it incurs adjustment costs, which increase at an increasing rate because resources have to be moved from producing output toward installing capital goods.\(^4\) Adjustment costs can be introduced into the above formulation by defining a function, \( \phi'(\alpha) \) where, \( \alpha = \Delta K/K \) is the growth rate of capital, \( \phi(0) = 1, \phi' > 0, \phi'' > 0 \), and \( \lim_{\alpha \to 0^+} \phi'(\alpha) = 0 \). With these changes the cost of investment, \( \Delta K \), becomes: \( p\phi(\alpha)\Delta K \) which is larger than it would be in the absence of adjustment costs, i.e. \( p\Delta K \). With these modifications (6) becomes:

\(^4\) In the absence of increasing marginal adjustment costs, the firm would change its stock of capital by finite amounts at an instant of time, implying infinite rates of investment. The existence of increasing marginal adjustment costs, however, provides the firm with an incentive to distribute its investment over time. Gould (1968), Lucas (1967), Uzawa (1969), and Mussa (1977) have emphasized the importance of increasing marginal adjustment costs for the derivation of a determinate investment function for the individual firm. See also Mirakhor (1993).
Maximizing (9) with respect to $\alpha$ yields the first order condition

$$\frac{r}{\rho} - \hat{\phi} \alpha - \hat{\phi} \leq 0$$

(10)

that says in order for investment to take place, the rate of profit expected, $r$, from that investment must be greater than the rate of return $\rho$, that the shareholders require in order to increase their equity shares in the firm; in other words, $q = \frac{r}{\rho} = \frac{V}{C} > 1$.

In the $q$ theory of investment, this result has important implications concerning the relationship between asset markets and the investment decisions of firms. On the physical side of the investment process, the firm is the demander of increments to its own stock of physical capital. On the financial side of the investment process, the firm is the supplier of the financial claims which correspond to its physical investment. To each unit of physical capital there corresponds a financial claim to the income stream which is generated by that unit of capital. The price which the firm can obtain for these claims is the implied market price for increment to the firm’s capital stock. From this perspective, the firm produces two products: final output and additional financial claims. Each of these products has a market price and the firm acts to maximize the augmented profit which it derives from the production and sale of these two products.

As was mentioned earlier, the critical variable in determining the firm’s investment behavior is marginal $q$ – the ratio of the shadow price of capital to its replacement cost – but the shadow price of capital is not observable. What one can observe is average $q$ – the ratio of market value of existing capital to its replacement cost. However, Hayashi (1982) has shown how the shadow price of capital – the market value of an additional unit of capital – is linked to the market valuation of existing capital. Hayashi’s analysis shows that under the assumptions of homogeneity of the production function and homogeneity of the capital adjustment cost function, the marginal value of capital equipment is equal to its average value, which may be measured using data on stock market valuation.

3.1 Application

To illustrate how the above ideas can be utilized to arrive at a value for cost of capital without resort to the rate of interest, define $V(t)$ as the value of current capital stock, i.e., the market value of the stock of capital evaluated by the
shareholders, \( Y(t) \) as the shareholders earning stream, and \( \rho \) as the rate of return expected by the shareholders, (2) becomes:

\[
V(t) = \int_0^t Y(t)e^{-\rho t} \, dt
\]

(11)

integration of (11) and rearrangement of terms yields

\[
\rho = \frac{Y}{V}; \ Y(t) = Y \text{ for all } t
\]

(12)

From (10) and (11), the supply price of capital is defined as the rate at which the shareholders are willing to supply the firm with financial resources. Equilibrium in equity market is achieved when the market value of equities, representing the quantity of capital demanded, equals the quantity held at the prevailing rate of return.

Costs of capital \( C(t) \) can be defined

\[
C(t) = \int_0^t Y_e(t)e^{-r t} \, dt
\]

(13)

where \( Y \) is the firm’s expected earning from capital goods and \( r \) is the expected rate of profit. Again, integrating (13) and rearranging terms yields:

\[
r = \frac{Y_e}{C}; \ Y_e(t) = Y_e \text{ for all } t
\]

(14)

which can be defined as the firm’s marginal efficiency of capital relative to its replacement cost. Assuming that the firm maximizes the shareholders’ wealth, otherwise they will not hold the firm’s equities, the firm can arrive at a decision rule regarding its investment plans by comparing (12) and (14). When the expectations of the shareholders and entrepreneurs are equal, then
using (15) and dividing (14) by (12) yields

\[
\frac{r}{\rho} = \frac{V}{C} = q
\]  

(16)

which is the same as equation (8). Tobin argued that if \( q \) exceeds unity, the value of capital investment would exceed its costs and the firm would have an incentive to invest. To calculate \( q \), we need to know \( V \) and \( C \). A simple method of calculating \( V \) is given by Gordon and Gould (1978) according to which\(^5\)

\[
V = \frac{(1 - b)Y}{(\rho - br)}
\]  

(17)

where:

- \( V = \) present value of the firm’s stock of capital. Since we are assuming that there are no debt instruments and the firm is equity financed only, this is also the value of the firm;
- \( b = \) expected value of the firm’s retention rate expressed as a fraction of the firm’s earnings;
- \( Y = \) expected value of the firm’s accounting earnings in the coming year;
- \( \rho = \) shareholders required rate of return or the yield at which the stock is selling. It is also the firm’s cost of equity capital; and
- \( r = \) firm’s expected rate of return on investment.

\(^5\) This is called the constant expected growth rate value model.
When investment is not only financed by retention but also by share of stock financing then, (17) becomes

\[ v = \frac{(1-d)Y}{r-d\rho} \]

(18)

where \( d = b+s \) and \( s \) is the expected rate of stock financing expressed as a ratio of the firm’s expected earnings.

From (12) and (15) we have:

\[ C = \frac{Y}{r} \]

(19)

substitution of (19) in (16) yields:

\[ r = \frac{qY}{V} \]

(20)

substitution of (20) in (18) yields:

\[ \rho = \frac{Y}{V}(1 - d + dq) \]

(21)

which relates the cost of equity capital to Tobin’s \( q \).

If a firm is already established, the required inputs into equation (21) can be obtained from the balance sheet of the firm. \( q \) can be calculated as a ratio of \( V \) to replacement costs. With no debt, \( V \) is the market value of the firm equal to the share holders’ equity valued at the year-end market price of the stock. Replacement costs include current assets, inventories, investment securities held by the firm, net property, land, and equipment. Having \( V \) and \( C \), \( q \) can be calculated. Additional
information needed are the accounting value of the firm’s expected earnings in the coming year (Y), the firm’s retention rate (b), and its expected stock financing rate (s), the last two expressed as a fraction of expected earnings.

3.2 A Numerical Example

Suppose for a given firm we have:

\[ V = $10 \text{ million}; C = $5 \text{ million}; Y = $0.70 \text{ million}, b = 0.35, s = 0.30, \text{ and } d = b+s = 0.35+0.30 = 0.65. \]  
For this firm \( q = 2 \) and

\[ \rho = \frac{Y}{V}(1-d+\frac{d}{q}) = \frac{0.7}{10}[1-(0.65)+(0.65)2] = 11.55\% \]

Therefore, the cost of capital for this firm is 11.55 percent which can then be compared with the rate of return to the investment project, i.e., the firm can compare \( \rho = 11.55\% \) to the expected rate of return \( r \), to determine if investment should take place. The desirability of an investment project is signaled through changes in \( q \) ratio while current share prices reflect expectations of future earnings of the firm.

If a firm is to be newly established and has no history of its own, \( C, V, \) and \( Y \) can be determined based on the data of the industry to which the firm belongs.

4. SUMMARY AND CONCLUSIONS

This paper has shown that it is possible to calculate cost of capital as a benchmark against which expected rates of return to projects can be measured in an economy where debt instruments do not exist and projects have to be equity financed, only by utilizing Tobin’s \( q \). The paper has presented the simplest model of \( q \) to derive a measure of cost of capital. Although more sophisticated models are available, the basic conclusion that the absence of interest rate mechanism need not deprive Islamic finance from having a benchmark for making efficient investment decisions remains valid. Moreover, as has been demonstrated extensively in the literature in recent years, it is possible to derive industry-wide or even economy-wide \( q \) s. As has been demonstrated recently by Choudhry and Mirakhor (1996) the concept of \( q \) can be used by Islamic governments to issue government securities to finance public projects.

REFERENCES


